Theory of Draw Resonance:

JAE CHUN HYUN

Technical Center
Monsanto Company
Pensacola, Florida 32575

Part I. Newtonian Fluids

Draw resonance in isothermal spinning is explained by using kinematic wave theory. The throughput wave equation and the expression for the throughput wave velocity are derived from the governing equations of the system, that is, the continuity, momentum, and constitutive equations. A comparison is made between twice the wave residence time and the thread-line residence time from the spinneret to the take-up in order to find the stable and draw resonance regions in terms of the drawdown ratio. For Newtonian fluids, the critical drawdown ratio to cause the onset of draw resonance is 19.744.

Part II will deal with draw resonance in the spinning of power law fluids and Maxwell fluids using the same wave theory. Calculation of the draw resonance amplitude when the drawdown ratio exceeds the critical values for Newtonian, power law, and Maxwell fluids will be explained in Part III.*

SCOPE

Melt spinning is the process in which fibers or films are produced by means of extruding, drawing, and cooling liquid into the form of filaments or sheets, respectively. Draw resonance is one of the major instabilities occurring in melt spinning as the drawdown ratio is increased and is manifested by a sustained periodic variation in the liquid cross-sectional area along the spin line.

This draw resonance phenomenon was first observed by Christensen (1962) and Miller (1963) and has since been investigated by many researchers experimentally (for example, Bergonzoni and DiCresce, 1966; Han et al., 1972; Zeichner, 1973; Donnelly and Weinberger, 1975; Ishihara and Kase, 1976) and theoretically (for example, Kase et al., 1966; Pearson and Matovich, 1969; Gelder, 1971; Shah and Pearson, 1972; Ishihara and Kase, 1975, 1976; Fisher and Denn, 1975, 1976).

A comprehensive review by Petrie and Denn (1976) treating the whole gamut of instabilities in polymer processing gives a full chronological picture of the draw resonance literature. Owing to these extensive efforts by various groups, the mechanics of draw resonance is understood in terms of its onset and response.

However, there remains the fundamental question of why this phenomenon does occur. The reason why this basic question has not been resolved so far is, in the author's view, mainly because stability analyses usually involve numerical calculations of stability equations (perturbations, eigenfunction approximations) with less emphasis on the physics of the system. The objective of this paper is thus to answer the above question by taking a new kinematic approach to draw resonance in isothermal melt spinning.

CONCLUSIONS AND SIGNIFICANCE

Although there is no propagating wave in melt spinning thread-lines which can be readily observed, there always exists an invisible kinematic wave which propagates from the spinneret to the take-up. That is the throughput wave (thread-line cross-sectional area times thread-line velocity) which is derived from the continuity equation. This throughput wave travels the spinning distance maintaining its constant amplitude with the unique wave velocity which is, in general, a function of drawdown ratio and

* To be published at a later date.

Jae Chun Yuan is presently with Tong Yang Cement Co., P.O. Box: Central 251, Seoul, Korea.

0001-1541-78-1017-0418-$00.75.

position on the thread-line (in the case of Newtonian fluids, a function of only the drawdown ratio).

Once we obtain the expression for the throughput wave velocity from the wave, *momentum, and constitutive* equations, it is straightforward to find the region of draw resonance instability in terms of the drawdown ratio. The crux of the idea is that in order for draw resonance (steady oscillation) to be established on the thread-line, two throughput waves (or disturbances), positive and negative deviations from the mean value, need to be able to travel the spinning distance from the spinneret to the take-up in a single thread-line residence time because the throughput passing through the system during one period of draw resonance is constant. Hence what we have to do in order to find the draw resonance region is just to compare twice the wave residence time with a thread-line residence time as the drawdown ratio is increased from a value of one.

The results of the comparison reveal that twice the wave residence time is larger than the thread-line residence *time if the drawdown ratio is smaller than* 19.744, *meaning* that the spinning is stable because disturbances are damped out owing to the lack of time. When the drawdown ratio is 19.744, the onset of draw resonance occurs because exactly two waves travel the spinning distance in one thread-line residence time and persists because of the constant take-up speed. When the drawdown ratio is larger than 19.744, the wave residence time remains equal to one half the thread-line residence time due to the increased draw resonance amplitude which increases the time averaged wave residence time over its steady state value, resulting in perpetuation of draw resonance. Details of the calculation of the draw resonance amplitude will be given in Part III.

Among the many instabilities encountered in polymer processing, draw resonance is an example where extensional flow dominates. Christensen (1962) and Miller (1963) first observed this phenomenon and named it as such. Subsequently, a number of papers have been published in the last decade dealing with both the experimental and theoretical aspects of the subject. Since there is a comprehensive review by Petrie and Denn (1976) which treats the whole gamut of polymer processing instabilities including draw resonance, we need only briefly mention several references which have some pertinence to our study.

While various workers have reported the occurrence of draw resonance in melt spinning of a number of polymers (for example, Bergonzoni and DiCresce, 1966; Han et al., 1972; Zeichner, 1973; Donnelly and Weinberger, 1975; Ishihara and Kase, 1976), several groups conducted theoretical studies using stability analysis techniques, for example, perturbations and eigenfunction approximations.

Two groups apparently were the first to attack the problem: Kase et al. (1966) and Pearson and Matovich (1969). Using infinitesimal perturbation methods, both groups analyzed isothermal spinning of Newtonian fluids in the absence of inertia, gravity, and surface forces and found 20.210 to be the critical drawdown ratio beyond which spinning becomes unstable. The same two groups later extended their analyses to power law fluids including secondary forces which were neglected earlier. (Ishihara and Kase, 1975, 1976; Pearson and Shah, 1974; Shah and Peterson, 1972.) Gelder (1971) calculated eigenvalues for isothermal spinning of Newtonian fluids by employing linearized approximate stability equations to confirm the value of 20.210 as the critical draw ratio. Recently, Fisher and Denn (1975, 1976) applied both linear and nonlinear stability analysis techniques (infinitesimal as well as finite-amplitude methods) to isothermal spinning of Newtonian fluids and Maxwell fluids. They confirmed the unique value of 20.210 for the Newtonian case and obtained a general plot of stability/instability regions for Maxwell fluids as functions of the drawdown ratio and relaxation time.

Owing to these extensive efforts by the above-mentioned people, certain aspects of the mechanics of draw resonance are now quite well understood. However, there remains the fundamental question of why this phenomenon does occur from a physical point of view. The reason why this basic question has not been resolved so far is, in the author's view, mainly because stability analyses usually involve numerical calculations of stability equations (perturbations, eigenfunction approximation) with less emphasis on the kinematics of the system. The objective of this paper is thus to try to answer that question by taking a new approach based on kinematics.

The tool we employ here to analyze draw resonance in isothermal melt spinning is the kinematic wave theory. There have been numerous applications of this theory in physical systems, for example, Courant and Friedricks (1948) in gas dynamics, Lighthill and Whitham (1955) in flood and traffic movements, Von Neumann and Richtmyer (1950) in hydrodynamics, and Rhee et al. (1970) in chromatography. Recently, Aris and Amundson (1973) wrote a comprehensive book to summarize and analyze wave theory from a chemical engineer's viewpoint. In the field of rheology, there are only a few publications about shear waves in viscoelastic fluids, that is, Coleman and Gurtin (1968) and Denn and Porteous (1971).

In this paper, we first derive a kinematic wave equation from the governing equations of isothermal melt spinning and then analyze and solve for draw resonance based on the wave equation.

DERIVATION OF THE THROUGHPUT WAVE EQUATION WITH THE EXPRESSION FOR THE THROUGHPUT WAVE VELOCITY

The governing equations for isothermal melt spinning of Newtonian fluids are as follows: if we neglect all the secondary forces on the thread-line, that is, inertia, gravity, and surface forces, and assume a uniform velocity distribution across the thread-line cross section, and the origin of the coordinate system starts at the die swell region, then [see Ishihara and Kase (1975) and Pearson and Matovich (1969]

Continuity equation:

$$\left(\frac{\partial A}{\partial t}\right)_x + \left[\frac{\partial (Av)}{\partial x}\right]_t = 0 \qquad (1)$$

Momentum equation:

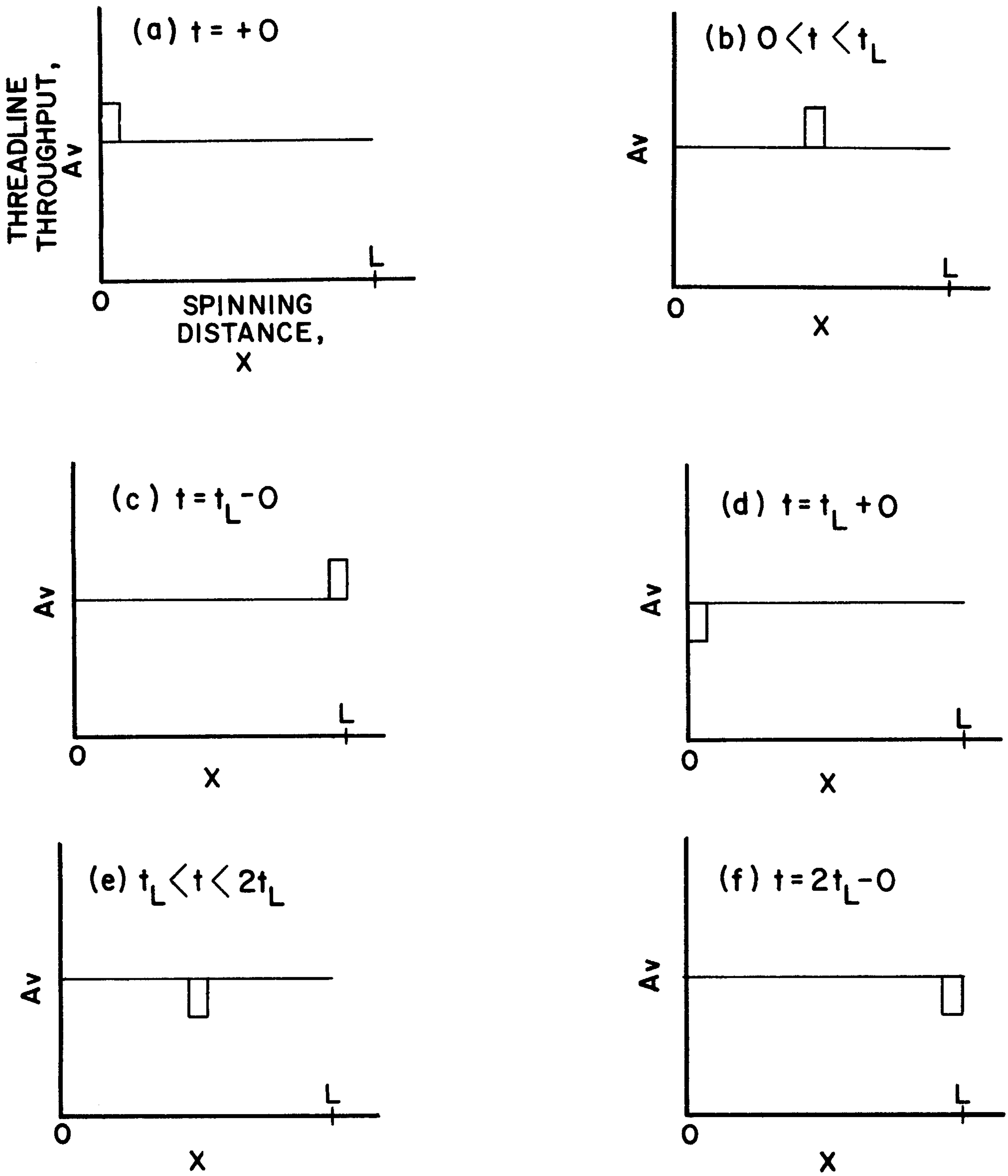

Fig. 1. A schematic illustrating the propagation of throughput waves.

$$\frac{\partial}{\partial x}\left[\mu A\left(\frac{\partial v}{\partial x}\right)_t\right] = 0 \quad (2)$$

The above subscripts x and t are used to clearly distinguish each partial derivative. Equation (1) can be rendered into the form of a wave equation

$$\left(\frac{\partial A}{\partial t}\right)_x + v\left(\frac{\partial A}{\partial x}\right)_t = -A\left(\frac{\partial v}{\partial x}\right)_t \quad (3)$$

where the thread-line cross-sectional area wave has the wave velocity of v and changes its value along the spin line (or the characteristics) according to

$$\frac{dA}{ds} = -A\left(\frac{\partial v}{\partial x}\right)_t \quad (4)$$

Equation (3) presents no more information about the system than (1) does because we still have to find the same expressions for v, A, and $(\partial v/\partial x)_t$. We multiply $[\partial(Av)/\partial A]_x$ through (1) to obtain

$$\left[\frac{\partial(Av)}{\partial t}\right]_x + \left[\frac{\partial(Av)}{\partial A}\right]_x\left[\frac{\partial(Av)}{\partial x}\right]_t = 0 \quad (5)$$

It is seen that Equation (5) is the throughput wave equation which has the wave velocity

$$U = \left[\frac{\partial (Av)}{\partial A} \right]_x \quad (6)$$

and maintains a constant value (amplitude) of the wave along the spin line (or the characteristics) because $d(Av)/ds = 0$.

This throughput wave thus exists on the thread-line, as (5) shows, although it can not be seen by an observer owing to the velocity term in the throughput expression, and propagates along the spinning distance with the velocity U given by (6). In fact, any throughput disturbance can become a throughput wave.

Next the throughput wave velocity expressed by (6) will be calculated as

$$U = \left[\frac{\partial (Av)}{\partial A} \right]_x = - \frac{\left[\frac{\partial (Av)}{\partial x} \right]_A}{\left(\frac{\partial A}{\partial x} \right)_{Av}} = - \frac{A \left(\frac{\partial v}{\partial x} \right)_A}{\left(\frac{\partial A}{\partial x} \right)_{Av}} \quad (7)$$

The denominator is a solution of steady state spinning where Av = constant = $A_o V_o = A_L V_L$ and

$$\frac{\partial A}{\partial t} = 0 \quad (8)$$

From the steady state solutions of (1) and (2) with $\partial A/\partial t = 0$, we get

$$A = A_o r^{-x/L}$$

$$v = v_o r^{x/L}$$

Hence

$$\left(\frac{\partial A}{\partial x} \right)_{Av} = - \frac{\ln r}{L} A \quad (9)$$

The substitution of (9) into (7) yields

$$U = \frac{L}{\ln r} \left(\frac{\partial v}{\partial x} \right)_A \quad (10)$$

The derivation of $(\partial v/\partial x)_A$ requires considering a fictitious process where A is held constant along the thread-line. The throughput Av in this case increases with x in order to satisfy the boundary conditions of $v = v_o$ at $x = 0$ and $v = rv_o$ at $X = L$. Then, since the thread-line force is always independent of x, we find $(\partial v/\partial x)_t$ also to be independent of x here:

$$\left[\because F = \mu A \left(\frac{\partial v}{\partial x} \right)_t \quad \text{from (2)} \right]$$

Next, because of the constant A, that is, $(\partial A/\partial x)_t = 0$, we have $(\partial v/\partial x)_t = (\partial v/\partial x)_A$ here, and consequently

$$\left(\frac{\partial v}{\partial x} \right)_A \quad (11)$$

is independent of x. The simple integration of the above along with the boundary condition yields

$$v = v_o + \left(\frac{\partial v}{\partial x} \right)_A x$$

and

$$\left(\frac{\partial v}{\partial x} \right)_A = \frac{v_o (r-1)}{L} \quad (12)$$

at the onset of draw resonance. The substitution of (12) into (10) gives the expression for the throughput wave velocity

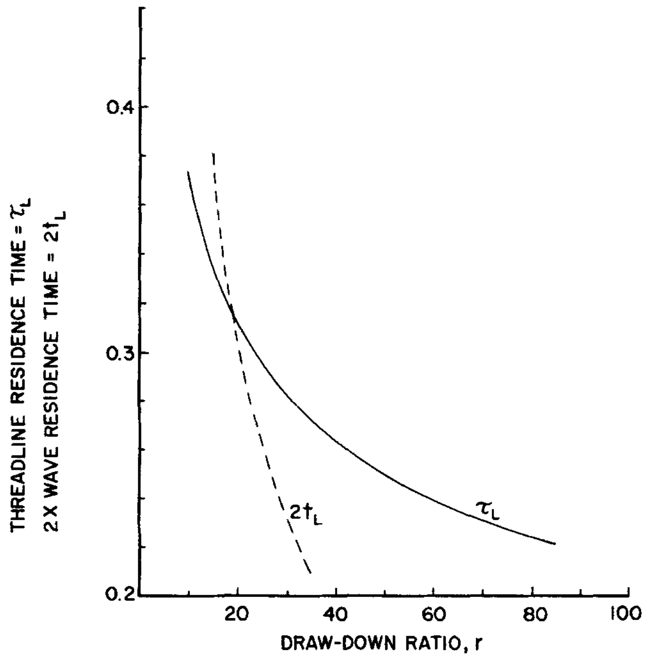

Fig 2. Wave residence time and threadline residence time vs. draw-down ratio for Newtonian fluids.

$$U = \frac{v_o (r-1)}{\ln r} \quad (13)$$

which is constant along the spin line for isothermal spinning of Newtonian fluids. The traveling time of a throughput wave (or disturbance) from the spinneret to the take-up is thus simply

$$t_L = \frac{L}{U} = \frac{L \ln r}{v_o (r-1)} \quad (14)$$

which is designated the throughput wave residence time.

KINEMATICS OF DRAW RESONANCE AND RESULTS

Before analyzing draw resonance, we must first consider how the throughput waves appear and continue once started. Any disturbances, whether they are originated at the spinneret, at the take-up, or at some point on the thread-line, propagate as throughput waves because they are bound to affect the throughput of the system which is otherwise constant along the thread-line. Of importance here is the fact that as soon as one throughput wave leaves the system through the take-up, another throughput wave with the opposite sign emerges at the spinneret and moves toward the take-up. This occurs because of the constant take-up speed which makes the thread-line tension force change in accordance with the throughput wave passing through the take-up. In other words, as Figure 1 shows, when a positive throughput wave goes by the take-up position, the thread-line tension force is increased because the thread-line tension is proportional to the thread-line cross-sectional area which becomes identical to the throughput wave divided by the constant speed at the take-up. This increased tension which is constant over the whole thread-line immediately causes an overattenuation of the thread-line at the spinneret, that is, a negative throughput wave. When this negative throughput wave passes by the take-up, a positive throughput wave starts at the spinneret again, by the same mechanism, and thus the cycle of throughput wave continues.

When does this cycle of throughput waves become a steady oscillation, that is, draw resonance, and when does it die out? In order for draw resonance to be established on the thread-line, a full cycle of throughput waves (or any multiple of the full cycle), a positive and a negative one, should travel through the thread-line in one thread-line residence time which is the time for the fluid to travel from the spinneret to the take-up. This is because the throughput, which comes into the system at a constant rate through the spinneret, must leave the system in a constant amount in every period of draw resonance.

The expression for the thread-line residence time (Lagrangian time at the take-up) can be obtained from the steady state solution as follows [see Hyun and Ballman (1978) for the detailed results of Lagrangian representations of isothermal melt spinning]:

$$\tau_L = \int_o^L \frac{dx}{v(x)} = \int_o^L \frac{dx}{v_o r^{x/L}} = \frac{L\left(1 - \frac{1}{r}\right)}{v_o \ln r} \tag{15}$$

From the discussion given above, the onset of draw resonance results when

$$2t_L = \tau_L \tag{16}$$

where t_L and τ_L are given by (14) and (15), respectively. As shown in Figure 2, Equation (16) is exactly satisfied when

$$r = 19.744\ldots \tag{17}$$

Accordingly, if $r < 19.744$, then $2t_L > \tau_L$, and any disturbance dies out owing to the lack of time for a full cycle of draw resonance to occur. At $r = 19.744$ we have $2t_L = \tau_L$, and thus the onset of draw resonance occurs. For $r > 19.744$ we would have $2t_L < \tau_L$ if the expressions for t_L and τ_L given by (14) and (15), respectively, hold. But when draw resonance occurs on the thread-line, the wave residence time t_L becomes larger than its steady state value given by (14) in proportion to the amplitude of draw resonance, while the thread-line residence time τ_L remains insensitive to the change of the amplitude of draw resonance. Hence, for $r > 19.744$, $2t_L = \tau_L$ maintains one cycle of draw resonance whose amplitude increases with r, until two cycles of draw resonance, that is, $4t_L = \tau_L$, start at $r = 77.84$. The details of the analysis of the draw resonance amplitude will be given in Part III.

To summarize, the value of 19.744 as the critical draw-down ratio causing the onset of draw resonance in isothermal spinning of Newtonian fluids has been derived by following an analytic route. The present analysis is extended to power law and Maxwell fluids in Part II.

ACKNOWLEDGMENT

The author is deeply indebted to R. L. Ballman for his support of this study and to the Monsanto Company for permission to publish this work.

NOTATION

A = thread-line cross-sectional area
F = thread-line tension force
L = distance from the spinneret to the take-up
r = drawdown ratio
s = parametric variable
t = time
t_L = throughput wave residence (traveling) time
U = throughput wave velocity
v = thread-line velocity
x = distance from the spinneret
μ = Newtonian extensional viscosity
τ_L = thread-line residence time

Subscripts

o = conditions at the spinneret
L = conditions at the take-up

LITERATURE CITED

Aris, R., and N. R. Amundson, *Mathematical Methods in Chemical Engineering, Volume 2. First Order Partial Differential Equations with Applications*, Prentice-Hall, Englewood Cliffs, N.J. (1973).

Bergonzoni, A., and A. J. DiCresce, "The Phenomenon of Draw Resonance in Polymeric Melts, Part 1. Qualitative View; Part 2. Correlation to Molecular Parameters," *Polymer Eng. Sci.*, **6**, 45 (1966).

Cristensen, R. E., "Extrusion Coating of Polypropylene," *S.P.E.J.*, **18**, 751 (1962).

Coleman, B. D., and M. E. Gurtin, "On the Stability Against Shear Waves of Steady Flows of Non-linear Viscoelastic Fluids," *J. Fluid Mech.*, **3**, 165 (1968).

Courant, R., and K. O. Friedricks, *Supersonic Flow and Shock Waves*, Interscience, New York (1948).

Denn, M. M., and K. C. Porteous, "Elastic Effects in Flow of Viscoelastic Liquids," *Chem. Eng. J.*, **2**, 280 (1971).

Donnelly, G. J., and C. B. Weinberger, "Stability of Isothermal Spinning of a Newtonian Fluid," *Ind. Eng. Chem. Fundamentals*, **14**, 334 (1975).

Fisher, R. J., and M. M. Denn, "Finite-Amplitude Stability and Draw Resonance in Isothermal Melt Spinning," *Chem. Eng. Sci.*, **20**, 1129 (1975).

——————, "A Theory of Isothermal Melt Spinning and Draw Resonance," *AIChE J.*, **22**, 236 (1976).

Gelder, D., "The stability of Fiber Drawing Processes," *Ind. Eng. Chem. Fundamentals*, **10**, 534 (1971).

Han, C. D., R. R. Lamonte, and Y. T. Shah, "Studies on Melt Spinning. III. Flow Instabilities in Melt Spinning: Melt Fracture and Draw Resonance," *J. Appl. Polymer Sci.*, **19**, 3291 (1972).

Hyun, J. C., and R. L. Ballman, "Isothermal Melt Spinning-Lagrangian and Eulerian Viewpoints," *Trans. Soc. Rheol.*, in press (1978).

Ishihara, H., and S. Kase, "Studies on Melt Spinning. V. Draw Resonance as a Limit Cycle," *J. Appl. Polymer Sci.*, **19**, 557 (1975).

——————, "Studies on Melt Spinning VI. Simulation of Draw Resonance Using Newtonian and Power-law Viscosities," *ibid.*, **20**, 169 (1976).

Kase, S., T. Matsuo, and Y. Yoshimoto, "Theoretical Analysis of Melt Spinning Part 2: Surging Phenomenon in Extrusion Casting of Plastic Films," *Sen-I KiKai GaKKaishi*, **19**, **T63** (1966).

Lighthill M. J., and G. B. Whitham, "On kinematic waves I. Flood movement in long rivers II. A theory of traffic flow on long crowded roads," *Proc. Roy. Soc. (London)*, **A229**, 281 (1955).

Miller, J. C., "Swelling Behavior in Extrusion," *S.P.E. Trans.* **3**, 134 (1963).

Pearson, J. R. A., and M. A. Matovich, "Spinning a Molten Threadline: Stability," *Ind. Eng. Chem. Fundamentals*, **8**, 605 (1969).

Pearson, J. R. A., and Y. T. Shah, "On the Stability of Isothermal and Nonisothermal Fiber Spinning of Power-law Fluids," *ibid.*, **13**, 134 (1974).

Petrie, C. J. S., and M. M. Denn, "Instabilities in Polymer Processing," *AIChE J.*, **22**, 209 (1976).

Rhee, H.-K., R. Aris, and N. R. Amundson, "On the theory of Multicomponent Chromatography," *Phil. Trans. Roy. Soc.*, **267**, 419 (1970).

Shah, Y. T., and J. R. A. Pearson, "Stability of Fiber Spinning of Power Law Fluids," *Polymer Eng. Sci.*, **12**, 219 (1972).

Von Neumann, J., and R. D. Richtmyer, "A Method for the Numerical Calculations of Hydrodynamic Shocks," *J. Appl. Phys.*, **21**, 232 (1950).

Zeichner, G., "Spinnability of Viscoelastic Fluids," M. ChE. thesis, Univ. Del., Newark (1973).

Manuscript received April 13, 1977; revision received December 8, and accepted December 16, 1977.